and Weglein[6] would assist in the evaluation, but would be considerably more complicated for the four-frequency type of operation.

Consequently, the authors feel that the theoretical results are most useful in predicting the type of operation which might be expected from a four-frequency parametric amplifier (and which might explain some inconsistencies in supposedly three-frequency operation). Meanwhile, the experimental results represent typical values obtained in the laboratory. Other configurations, particularly those which permit more independent control of tuning and loading of each of the four frequencies, and which perhaps allow some measure of independent control of the diode coefficients C_1 and C_2, may produce superior results and may be easier to correlate with theory.

Further experiments with the crossed-guide and circulator type paramps mentioned above have shown that for best noise performance the amplifier adjustments seem to produce a regenerative gain at the signal input port of about 8 db, when the up-converter gain is about 20 db. The difference of 12 db is within 2 db of the theoretical gain due to up-conversion alone. This appears to indicate that best noise performance is obtained in the potentially-unstable region of negative input conductance, in Figs. 4 and 6. Correlation with theory would have to be made in terms of transducer gain, as in (22). Because of the negative input conductance, it has been found very necessary to operate these amplifiers with an isolator on the signal input port to provide stability against input mismatch over the entire frequency band of the amplifier, particularly for use with a radar having many wavelengths of transmission line between the actual antenna and the parametric amplifier.

[6] R. C. Knechtli and R. D. Weglein, "Low-noise parametric amplifier," Proc. IRE, vol. 48, pp. 1218–1226; July, 1960.

V. Acknowledgment

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Ferrites with Planar Anisotropy at Microwave Frequencies*

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Summary—Materials with an easy plane of magnetization (planar anisotropy) have recently been discovered. The large anisotropy field that tends to keep the magnetization in the easy plane reduces the field required to cause ferromagnetic resonance, which makes the material promising for microwave applications. Equations are derived for the susceptibility, taking into account losses and a finite medium. Propagation in a longitudinal and transverse static field is considered. The location of a slab in a rectangular waveguide for minimum loss in the forward direction, and the use of the material as a phase shifter, are discussed. Experimental microwave data on some materials are given, and also data on an isolator and phase shifter incorporating these materials.

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I. Introduction

For most ceramic magnetic materials used in the microwave frequency range, the magnetic anistropy field is sufficiently small so that it can be neglected in deriving equations for engineering applications. However, there are two groups of materials that have very large anisotropy fields that cannot be neglected. Both of these groups have a hexagonal crystal structure; the first has an easy direction of magnetization along the C axis, and the second has an easy plane of magnetization perpendicular to the C axis (planar anisotropy). The first group includes the ceramic permanent magnetic material, barium ferrite, marketed under various trade names such as Ferroxdure, Magnadur, Indox, Ceramagnets, etc. The latter group is called Ferroxplana and has been discovered more recently. Both of these materials are described in the new book by Smit and Wijn[1] which itself contains numerous references.

Some articles describing microwave applications of ceramic permanent magnet materials have appeared in the literature;[2-5] however, though there have been articles discussing the ferromagnetic resonance of ferrites with planar anistotropy[6,7] the author is not aware of any publication discussing the microwave applications of these ferrites. It is the purpose of this paper to develop some general equations for the microwave applications of ferrites with planar anistropy and to present some experimental data on the material and devices incorporating this material. It will be assumed that the materials are polycrystalline and are oriented so that all easy planes of magnetization are mutually parallel. The only magnetic anisotropy field that will be considered will be the one that tends to hold the magnetization vector in the easy plane. The term planar ferrites will frequently be used as a short way of saying ferrites with planar anisotropy.

II. Equations of Magnetization and Susceptibility: Applied Field in Easy Plane

A. Infinite Medium, No Losses

In the absence of anistropy fields, the equation of magnetization is simply

$$\frac{1}{\gamma}\frac{d\overline{M}}{dt} = \overline{M} \times \overline{H},$$

where γ is the gyromagnetic ratio, $\overline{M}$ the magnetization vector, and $\overline{H}$ the magnetic field. The quantity $\overline{M} \times \overline{H}$ constitutes a torque. The anistropy field of the planar ferrites produces an additional torque $\overline{T}_a$ that tends to keep magnetization vector in the easy plane. The equation of magnetization now becomes

$$\frac{1}{\gamma}\frac{d\overline{M}}{dt} = \overline{M} \times \overline{H} + \overline{T}_a. \qquad (1)$$

[1] J. Smit and H. P. Wijn, "Ferrites," John Wiley and Sons, New York, N. Y.; 1959.

[2] M. T. Weiss and F. A. Dunn, "A 5-mm resonance isolator," IRE Trans. on Microwave Theory and Techniques, vol. 6, p. 331; July, 1958.

[3] M. T. Weiss, "The behavior of Ferroxdure at microwave frequencies," 1955 IRE Convention Record, vol. 3, pt. 8, p. 95.

[4] D. J. DeBitetto, F. K. DuPre, and F. G. Brockman, "Highly Anistropic Materials for Millimeter Wave Applications," presented at the Symp. on Millimeter Waves, Polytechnic Inst. of Brooklyn, New York; March 31–April 2, 1959.

[5] L. C. Kravitz and G. S. Heller, "Resonance isolator at 70 kMc," Proc. IRE, vol. 47, p. 331; February, 1959.

[6] H. S. Belson and C. J. Kriessman, "Microwave resonance in hexagonal ferromagnetic single crystals," *J. Appl. Phys.*, supplement to vol. 30, no. 4, p. 175S; April, 1959.

[7] E. Schlömann and R. V. Jones, "Ferromagnetic resonance in polycrystalline ferrites with hexagonal crystal structure," *J. Appl. Phys.*, supplement to vol. 30, no. 4, p. 177S; April, 1959.

It is now proposed to develop the equation for $\overline{T}_a$ and to expand (1) in Cartesian coordinates. (See Fig. 1.) The easy plane of magnetization will be taken as the XY plane.

The anisotropy energy of materials with one axis of symmetry is given by[8]

$$E_a = K_1 \sin^2\theta + K_2 \sin^4\theta \cdots. \qquad (2)$$

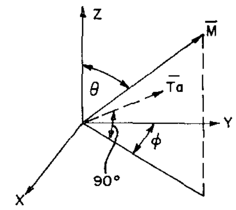


Fig. 1—Representation of magnetization and torque vectors.

The torque caused by this anisotropy is equal to minus the derivative of the energy. Thus,

$$\overline{T}_a = -(2K_1 \sin\theta\cos\theta + 4K_2 \sin^3\theta\cos\theta)\hat{\theta},$$

where the symbol ^ over a letter denotes a unit vector.

In this paper we will consider only cases where the magnetization will be in, or very close to, the easy plane; *i.e.*, $\theta \approx \pi/2$. Therefore,

$$\sin\theta \approx 1, \qquad \cos\theta \approx \frac{M_z}{M_s};$$

$$\overline{T}_a = -(2K_1 + 4K_2)\frac{M_z}{M_s}\hat{\theta}.$$

The magnetic anisotropy field is defined as[9]

$$H_a = -\frac{2(K_1 + 2K_2)}{M_s}.$$

We thus have

$$\overline{T}_a = H_a M_z \hat{\theta} \qquad (3)$$

where $\hat{\theta}$ is in the XY plane and is perpendicular to $\overline{M}_s$. It points in the direction a right-hand screw moves in turning $\overline{M}$ towards the XY plane. Thus, $\hat{\theta} = -\cos\phi\hat{x} + \sin\phi\hat{y}$ where ϕ is the angle between the XY component of $\overline{M}$ and the torque. For the case where θ is almost equal to $\pi/2$, we can write

$$\cos\phi = \frac{M_y}{M_s}; \qquad \sin\phi = \frac{M_x}{M_s}. \qquad (4)$$

Eq. (3) thus becomes

$$\overline{T}_a = \frac{H_a M_z}{M_s}(-M_y\hat{x} + M_x\hat{y}). \qquad (5)$$

[8] Smit and Wijn, *op. cit.*, p. 48.

[9] *Ibid.*, p. 49.

We now assume that we apply a static biasing field H_0 in the Y direction and saturate the material in that direction. Thus, we can take M_y as approximately equal to M_s. The applied RF magnetic fields will be taken as lying in the XZ planes. Thus, the $\overline{M}$ and $\overline{H}$ of (1) can be written as

$$\overline{M} = (M_s\hat{y} + m_x\hat{x} + m_z\hat{z})$$

$$\overline{H} = (H_0\hat{y} + h_x\hat{x} + h_z\hat{z}).$$

Note that dc components here and in the remainder of this paper will be expressed in capital letters, whereas RF components will be expressed in small letters.

Eq. (1) can now be written as follows:

$$\frac{j\omega m_x}{\gamma} = M_s h_z - m_z H_0 - H_a m_z \qquad (6a)$$

$$\frac{j\omega m_y}{\gamma} = m_z h_x - m_x h_z + H_a m_z \frac{m_x}{M_s} \qquad (6b)$$

$$\frac{j\omega m_z}{\gamma} = -M_s h_x + m_x H_0. \qquad (6c)$$

Eq. (6b) can be omitted from further consideration since the quantities involved are much smaller than the quantities in the other two equations. Eqs. (6a) and (6c) can now be solved as follows:

$$m_x = \frac{M_s}{D_1}\left[(H_0 + H_a)h_x - jH_i h_z\right]$$

$$m_z = \frac{M_s}{D_1}\left[jH_i h_x + H_0 h_z\right] \qquad (7)$$

where

$$D_1 = H_0(H_0 + H_a) - H_i^2$$

$$H_i = \frac{\omega}{|\gamma|}.$$

The flux $\bar{b}$ is given by the equation

$$\bar{b} = \bar{h} + 4\pi\bar{m} = \bar{h} + \begin{vmatrix} \chi_{xx} & -j\chi_{xz} & 0 \\ j\chi_{xz} & \chi_{zz} & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} h_x \\ h_z \\ h_y \end{vmatrix}$$

$$= (1 + [\chi])\bar{h} \qquad (8)$$

$$\chi_{xx} = \frac{4\pi M_s}{D_1}(H_0 + H_a)$$

$$\chi_{zz} = \frac{4\pi M_s}{D_1} H_0$$

$$\chi_{xz} = \frac{4\pi M_s}{D_1} H_i. \qquad (9)$$

We note that the H_0 required for resonance is much smaller than in the case of conventional ferrites, and that $\chi_{xx} \neq \chi_{zz}$.

The extent to which the anisotropy field decreases the magnitude of the external field required for resonance is illustrated in Table I. (γ is taken as 2.8 Mc per oersted.) The advantage of this for microwave devices is obvious.

TABLE I

Frequency	H_a	Field for Resonance	
(Mc)	(oe)	Isotropic (oe)	Planar (oe)
3000	5000	1070	230
5000	5000	1780	580
10,000	5000	3580	1800
5000	10,000	1780	330
10,000	10,000	3580	1100
20,000	10,000	7160	3700

B. Finite Medium, No Losses

Assuming that the sample is in the shape of an ellipsoid, demagnetizing factors N_x, N_y, N_z can be introduced. Eq. (6) can then be rewritten as

$$\frac{j\omega m_x}{\gamma} = M_s(h_z - N_z m_z) - m_z(H_0 - N_y M_s) - H_a m_z$$

$$\frac{j\omega m_z}{\gamma} = -M_s(h_x - N_x m_x) + m_x(H_0 - N_y M_s). \qquad (10)$$

The terms in the tensor relating magnetization to externally applied fields are

$$\chi_{xx} = \frac{4\pi M_s H_x}{D_2}$$

$$\chi_{zz} = \frac{4\pi M_s H_z}{D_2}$$

$$\chi_{xz} = \frac{4\pi M_s H_i}{D_2}, \qquad (11)$$

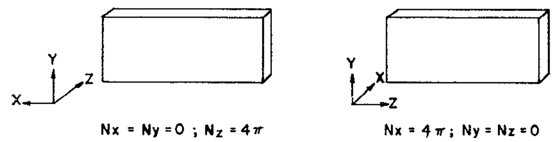
where

$$H_x = H_0 + H_a + M_s(N_z - N_y); \quad D_2 = H_x H_z - H_i^2$$

$$H_z = H_0 + M_s(N_x - N_y); \quad H_i = \frac{\omega}{|\gamma|}.$$

From (11) we note that the anisotropy field operates in the same way as the fields caused by the inequality of the demagnetizing factors. However, the anistropy field may be as high as 30,000 oersteds, whereas the field caused by the inequality of demagnetizing factors cannot exceed the saturation magnetization, which can be as high as 6000 with present materials.

The demagnetizing fields caused by the inequality of the demagnetizing factors can of course be used to supplement the anisotropy field in reducing the external biasing field required for resonance. Two instances where this is accomplished are shown in Fig. 2, where it is assumed that the slab is very thin. The greatest reduction in external field required will be for the slab in Figure 2(b). Thus, for a material with an anisotropy field of 10,000 oersteds and a saturation magnetization of 2000 oersteds, the fields required for resonance at 15,000 Mc would be 600, 2000, and 2200 oersteds for the geometry shown in Fig. 2(b), for the geometry shown in Figure 2(a), and for the infinite medium, respectively. In an infinite isotropic medium the field required would be 5300 oersteds.



(a) (b)

Fig. 2—Planar ferrite slabs. (a) Easy plane in plane of slab. (b) Easy plane perpendicular to plane of slab.

C. *Infinite Medium, With Losses*

Losses (or damping) will be introduced, through the Landau-Lifshitz formulation. From (1), we note that there are two torque terms to be considered. The first is $\overline{M}\times\overline{H}$. The loss term corresponding to this torque will be a vector that is proportional to this torque and is perpendicular to both the torque and the magnetization vector. The loss term for this torque can thus be represented as

$$\frac{\alpha\overline{M}}{M_s}\times(\overline{M}\times\overline{H}),$$

where α is the proportionality constant. This can be expanded to give

$$\alpha[(H_0m_x - M_sh_x)\hat{x} + (H_0m_z - M_sh_z)\hat{z}].$$

The second torque term is $-H_am_z\hat{x}$. (See (5); the term $M_x\hat{y}$ can be neglected.) The loss term for this torque can thus be represented by

$$-\frac{\alpha H_am_z}{M_s}(\overline{M}_x\,\hat{x}) = \alpha H_am_z\hat{z}.$$

The equation of magnetization thus becomes

$$\frac{j\omega m_x}{\gamma} = M_sh_z - m_zH_0 - H_am_z + \alpha(H_0m_x - M_sh_x)$$
$$\frac{j\omega m_z}{\gamma} = m_xH_0 - M_sh_x + \alpha(H_0m_z - M_sh_z + H_am_z). \quad (12)$$

The terms of the susceptibility tensor for this case are shown in (13). The single prime denotes the reactive part and the double prime the dissipative part of the complex term; *i.e.*, $\chi^* = \chi' - j\chi''$.

$$\chi'_{xx} = \frac{4\pi M_s}{D_3}\{[1+\alpha^2][H_0+H_a][(1+\alpha^2)(H_0+H_a)H_0 - H_i^2] + \alpha^2(2H_0+H_a)H_i^2\}$$
$$\chi''_{xx} = \alpha\frac{4\pi M_sH_i}{D_3}\{(1+\alpha^2)(H_0+H_a)^2 + H_i^2\}$$
$$\chi'_{zz} = \frac{4\pi M_s}{D_3}\{[1+\alpha^2]H_0[(1+\alpha^2)(H_0+H_a)H_0 - H_i^2] + \alpha^2(2H_0+H_a)H_i^2\}$$
$$\chi''_{zz} = \alpha\frac{4\pi M_sH_i}{D_3}\{(1+\alpha^2)H_0^2 + H_i^2\}$$
$$\chi'_{xz} = \frac{4\pi M_sH_i}{D_3}\{(1+\alpha^2)(H_0+H_a)H_0 - H_i^2\}$$
$$\chi''_{xz} = \alpha\frac{4\pi M_s}{D_3}\{H_i^2(2H_0+H_a)\}$$
$$D_3 = [(1+\alpha^2)(H_0+H_a)H_0 - H_i^2]^2 + [\alpha H_i(2H_0+H_a)]^2$$
$$H_i = \frac{\omega}{|\gamma|}. \quad (13)$$

D. *Finite Medium, With Losses*

Taking into consideration both a finite ellipsoid and losses, the terms in the susceptibility tensor are shown in (14). Terms in α^2 have been omitted in this equation.

$$\chi'_{xx} = \frac{4\pi M_s}{D_4}H_x(H_xH_z - H_i^2)$$
$$\chi''_{xx} = \frac{\alpha 4\pi M_sH_i}{D_4}[H_x(H_x+H_z) - (H_xH_z - H_i^2)]$$
$$\chi'_{zz} = \frac{4\pi M_s}{D_4}H_z(H_xH_z - H_i^2)$$
$$\chi''_{zz} = \frac{\alpha 4\pi M_sH_i}{D_4}[H_z(H_x+H_z) - (H_xH_z - H_i^2)]$$
$$\chi'_{xz} = \frac{4\pi M_s}{D_4}H_i[H_xH_z - H_i^2]$$
$$\chi''_{xz} = \frac{\alpha 4\pi M_sH_i^2}{D_4}[H_x+H_z]$$
$$D_4 = [H_xH_z - H_i^2]^2 + [\alpha H_i(H_x+H_z)]^2. \quad (14)$$

H_x, H_z, and H_i have been defined in connection with (11).

E. *Linewidth*

Linewidth can be defined as the difference in the two values of the biasing field at which the loss term of the susceptibility is one-half its maximum value. (For the purposes of this discussion any of the terms χ''_{xx} X''_{zz}, or X''_{xz}, may be used.) For materials in which α is very much less than 1, the loss term of the susceptibility will be one-half of its maximum value when the two terms in the denominator D_3 of (13) are equal. Disregarding terms in α^2 we thus have (see Fig. 3),

$$H_{0_{1/2}}(H_{0_{1/2}} + H_a) - H_i^2 = \alpha H_i(2H_{0_{1/2}} + H_a)$$

$$\text{Let } H_{0_{1/2}} = H_{0r} \pm \Delta H_{1/2}.$$

Disregarding second-order terms, we get $\Delta H_{1/2} = \alpha H_i$, or $\Delta H = 2\alpha H_i$ where H_i is the field at which an isotropic ferrite would be resonant.

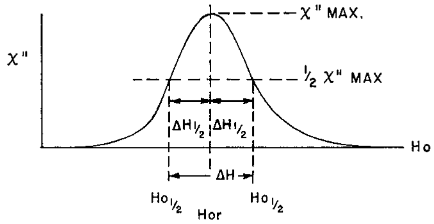


Fig. 3—Resonance curve.

The conclusion thus is that the linewidth for the isotropic ferrite and planar ferrites will be the same. In general, the entire theoretical resonance-loss curve of planar ferrites is very similar to that of the isotropic ferrites.

It is also of interest to compare the maximum of the loss-term of susceptibility in the two cases. In the case of isotropic ferrites, we have (disregarding terms in α^2),

$$\chi''_{\max} = \frac{4\pi M_s}{2\alpha H_i}.$$

In the case of planar ferrites we have

$$\chi''_{xx_{\max}} = \frac{4\pi M_s}{\alpha H_i} \cdot \frac{(H_0 + H_a)^2 + H_i^2}{(2H_0 + H_a)^2}.$$

For $H_a \gg H_0$, we have

$$\chi''_{xx_{\max}} = \frac{4\pi M_s}{\alpha H_i} = 2\chi''_{\max}.$$

Thus, the maximum value of susceptibility χ_{xx}'' in the case of planar ferrites may double that in the case of isotropic ferrites. However, the maximum value of χ_{zz}'' will be very much smaller, as shown below:

$$\frac{\chi''_{xx_{\max}}}{\chi_{zz_{\max}}} = \frac{(H_0 + H_a)^2 + H_0(H_0 + H_a)}{H_0^2 + H_0(H_0 + H_a)} = \frac{H_0 + H_a}{H_0}. \quad (15)$$

III. Applied Field Perpendicular to Easy Plane

If the biasing field is applied perpendicular to the easy plane of magnetization, and if its magnitude is less than the anisotropy field H_a, the material will not be saturated, and the type of analysis applied above is completely inapplicable. There will very likely be domains in the easy plane, and susceptibility will then be caused by domain wall motion. Moreover, the anisotropies in the basal plane will now be important and complicate the picture further. There may possibly be some useful applications for such a case, but it is not intended to analyze this case in this paper.

If the biasing field is perpendicular to the easy plane and its magnitude exceeds the anisotropy field, then the material will be saturated and a straightforward analysis can be made. However, it is not intended to make this analysis in this report, as the principal attraction of planar ferrites for linear applications in the microwave region appears to lie in the fact that, by virtue of its anisotropy, the field required for resonance or other applications is much smaller than in isotropic ferrites. In the above case, the field for resonance will be much greater than that required for isotropic ferrites.

IV. Plane-Wave Propagation

A. Longitudinal Magnetization

1) *Determination of Phase Constants for Plane Waves:* From Maxwell's equations, we readily obtain

$$\nabla \times \nabla \times \bar{h} = \omega^2 \epsilon \mu_0 \bar{h}(1 + [\chi]). \quad (16)$$

The easy plane of magnetization will be taken in the XY plane, and the biasing field applied in the Y direction. Assume a plane wave propagating in the Y direction with a propagation constant $j\beta$. (Assume the medium to be lossless.) Then

$$\frac{\partial}{\partial_x} = \frac{\partial}{\partial_z} = 0$$

$$\nabla \times \nabla \times \bar{h} = \hat{x}\beta^2 h_x + \hat{z}\beta^2 h_z. \quad (17)$$

Combining (16) and (17) we get

$$\beta^2 h_x = \beta_d^2[h_x(1 + \chi_{xx}) + j\chi_{xz}h_z]$$
$$\beta^2 h_z = \beta_d^2[h_z(1 + \chi_{zz}) - j\chi_{xz}h_x], \quad (18)$$

where $\beta_d = \omega^2 \epsilon \mu_0$.

Solving for β^2 by setting the determinant of (18) to zero, yields,

$$\beta_+^2 = \beta_d^2 \left\{1 + \frac{1}{2} \frac{4\pi M_s}{H_0(H_0 + H_a) - H_i^2} \cdot [2H_0 + H_a + (H_a^2 + 4H_i^2)^{1/2}]\right\}$$

$$\beta_-^2 = \beta_d^2 \left\{1 + \frac{1}{2} \frac{4\pi M_s}{H_0(H_0 + H_a) - H_i^2} \cdot [2H_0 + H_a - (H_a^2 + 4H_i^2)^{1/2}]\right\} \quad (19)$$

where

$$H_i = \frac{\omega}{|\gamma|}.$$

For a numerical example, let us take the following values:

$f = 10{,}000$ Mc $\quad H_0 = 300$ oersteds
$4\pi M_s = 2000$ gauss $\quad H_a = 10{,}000$ oersteds

We then obtain

$$\beta_+ = -1.3\beta_d; \qquad \beta_- = 1.16\beta_d.$$

If a conventional ferrite with the same $4\pi M_s$ value were used, the values obtained would be $\beta_+ = 0.39\beta_d$; $\beta_- = 1.5\beta_d$. A biasing field of about 2700 and 9000 oersteds would be required for β_+ and β_- respectively to reach the values given for the case of the planar ferrites.

2) *Normal Modes*: In conventional ferrites, the normal modes of propagation are the circularly polarized modes, *i.e.*, $h_x = \pm jh_z$, as has been shown in the literature. The normal modes of propagation in the case of planar ferrites can be obtained by substituting the values of β given in (19) into (18), thereby obtaining the corresponding ratios of h_z and h_x:

$$\left(\frac{h_z}{h_x}\right)_\pm = \frac{-j}{2H_i}\left[H_a \mp (H_a^2 + 4H_i^2)^{1/2}\right]. \tag{20}$$

We note, by multiplying out, that

$$\left(\frac{h_z}{h_x}\right)_+ \cdot \left(\frac{h_z}{h_x}\right)_- = 1.$$

Thus we see that the normal modes in planar ferrites are elliptically polarized, and the ratios of h_z and h_x for the two modes are reciprocal. It can be shown that if h_z and h_x have the ratios indicated in (20), the relation between $\bar{b}$ and $\bar{h}$ is scalar; *i.e.*, $\bar{b} = \mu_\pm \bar{h}$ where $\mu_\pm$ are the values in the parentheses in (19).

It can be readily shown that it is not possible to have Faraday rotation with planar ferrites. Faraday rotation requires that there be circular symmetry in the plane perpendicular to the direction of propagation. In the case of planar ferrites, this is manifestly not true. The nonexistence of Faraday rotation can be shown explicitly by considering a plane-polarized wave existing at one point along the direction of propagation. Numerical computation will show that the wave will be elliptically polarized at all other points except for discrete points at regular intervals along the direction of propagation.

B. Transverse Magnetization

There are two cases to consider, depending on whether the direction of propagation is perpendicular to the plane of easy magnetization, or whether it is in the plane. Consider the former first; *i.e.*, the XY plane is the easy plane, H_0 is in the Y direction, and propagation is in the Z direction. Then $\partial/\partial x = \partial/\partial y = 0$, and $\partial/\partial z = -j\beta$, where $j\beta$ is the propagation constant:

$$\nabla \times \nabla \times \bar{h} = \hat{x}\beta^2 h_x.$$

Combining (16) and (20) we get

$$\beta^2 h_x = \beta_d^2[h_x(1 + \chi_{xx}) + j\chi_{xz}h_z]$$
$$0 = \beta_d^2[h_z(1 + \chi_{zz}) - j\chi_{xz}h_x]$$
$$\beta^2 = \beta_d^2\left[\frac{(1 + \chi_{xx})(1 + \chi_{zz}) - \chi_{xz}^2}{1 + \chi_{zz}}\right]. \tag{21}$$

For propagation in the X direction we get

$$\beta^2 = \beta_d^2\left[\frac{(1 + \chi_{xx})(1 + \chi_{zz}) - \chi_{xz}^2}{1 + \chi_{xx}}\right]. \tag{22}$$

The quantities in brackets in (21) and (22) define an effective relative permeability for the medium for the two cases considered above.

V. Microwave Applications

A. Resonance Isolator: Location of Slab in Rectangular Waveguide for Minimum Absorption at Resonance for One Direction of Propagation

In the following discussion it is assumed that the volume of the slab is small, so that perturbation theory is applicable. Starting with (10), we readily derive the following:

$$\frac{m_x}{M_s} = \frac{(H_0 + R)h_x - jH_ih_z}{(H_0 + R)(H_0 + S) - H_i^2}$$
$$\frac{m_z}{M_s} = \frac{jH_ih_x + (H_0 + S)h_z}{(H_0 + R)(H_0 + S) - H_i^2} \tag{23}$$
$$R = H_a + M_s(N_z - N_y); \quad H_i = \frac{\omega}{|\gamma|}$$
$$S = M_s(N_x - N_y).$$

The denominator in (23) will equal zero at a value of H_0 given in (24):

$$H_0 = -\frac{R + S}{2} + \frac{1}{2}\left[(R + S)^2 - 4(RS - H_i^2)\right]^{1/2}. \tag{24}$$

For the above value of H_0, m_x and m_z will tend to be extremely large. However, for a particular ratio of h_x to h_z, the numerators of (23) will be zero. Let b be defined so that $h_z = -jbh_x$. Setting the numerator of either of the equations in (23) to zero, and substituting the value of H_0 given in (24), we get

$$b = \frac{1}{2H_i}\left[\sqrt{(R + S)^2 - 4(RS - H_i^2)} + (R - S)\right]. \tag{25}$$

Assuming a thin slab in the configuration of Fig. 4(a), we have $N_x = N_y = 0$; $N_z = 4\pi$;

$$b = \frac{1}{2H_i}[H_a + 4\pi M_s + \sqrt{(H_a + 4\pi M_s)^2 + 4H_i^2}]. \quad (26)$$

As a numerical example, let $H_a = 10{,}000$ oersteds, $4\pi M_s = 3000$ gauss, with an operating frequency of 15,000 Mc. The value of b comes out to be 2.78. The distance d can now be readily determined by finding the location in the (unperturbed) waveguide at which h_z and h_x have the desired ratio.

If $H_a = 0$, then $b = 1.31$. Thus, in the case of planar ferrites, the location of the slab is considerably closer to the center of the waveguide than in the case of isotropic ferrites.

Consider next the slab shown in Fig. 4(b). The easy plane is perpendicular to the plane of the slab. Eq. (25) applies in this case, too. However, whereas in the first case, b is the ratio of transverse to longitudinal RF magnetic fields, in the second case, b is the ratio of the longitudinal to transverse fields. For a thin slab, $N_x = 4\pi$, $N_y = N_z = 0$. Eq. (25) now becomes

$$b = \frac{1}{2H_i}[H_a - 4\pi M_s + \sqrt{(H_a - 4\pi M_s)^2 + 4H_i^2}]. \quad (27)$$

The location of the slab for a resonance isolator will now be closer to the narrow wall than in the case of isotropic ferrites.

The third and fourth cases are illustrated in Figs. 4(c) and 4(d). Eq. (25) applies, $N_x = N_z = 0$; $N_y = 4\pi$;

$$b = \frac{1}{2H_i}[H_a + \sqrt{H_a^2 + 4H_i^2}]. \quad (28)$$

As in the case of isotropic ferrites, the ratio of transverse to longitudinal h fields for minimum absorption is the same as in the infinite medium.

It is of interest to consider how the location for minimum absorption at resonance for the third and fourth cases varies as a function of frequency. To do this, it is necessary to decompose the RF magnetic fields in the waveguide into the normal modes given in (20):

$$h_x + jh_z = P(b + j) + Q(1 - jb).$$

$(b+j)$ corresponds to the plus sign in (20) and will be designated as positive polarization; $(1-jb)$ then corresponds to negative polarization:

$$P = \frac{bh_x + h_z}{1 + b^2}, \qquad Q = \frac{h_x - bh_z}{1 + b^2}.$$

A plot of the distribution of positive and negative polarization (P and Q) as a function of frequency is shown in Fig. 5.[10] H_a was taken as 10,000 oersteds; all the curves have been normalized so that $Q = 1$ when $P = 0$.

[10] For isotropic ferrites compare Fig. 26 in, A. G. Fox, S. E. Miller and M. T. Weiss, "Behavior and applications of ferrites in the microwave region," *Bell Syst. Tech. J.*, vol. 34, p. 5; January, 1955.

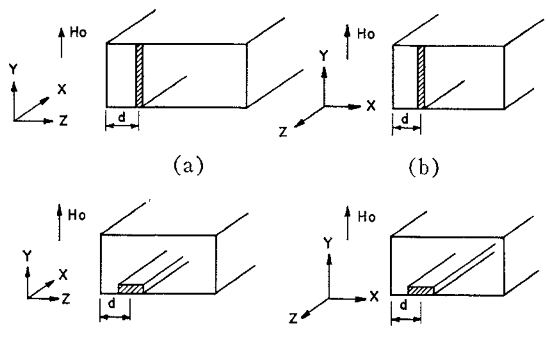


Fig. 4—Planar ferrite slabs in rectangular waveguide; XY plane is easy plane.

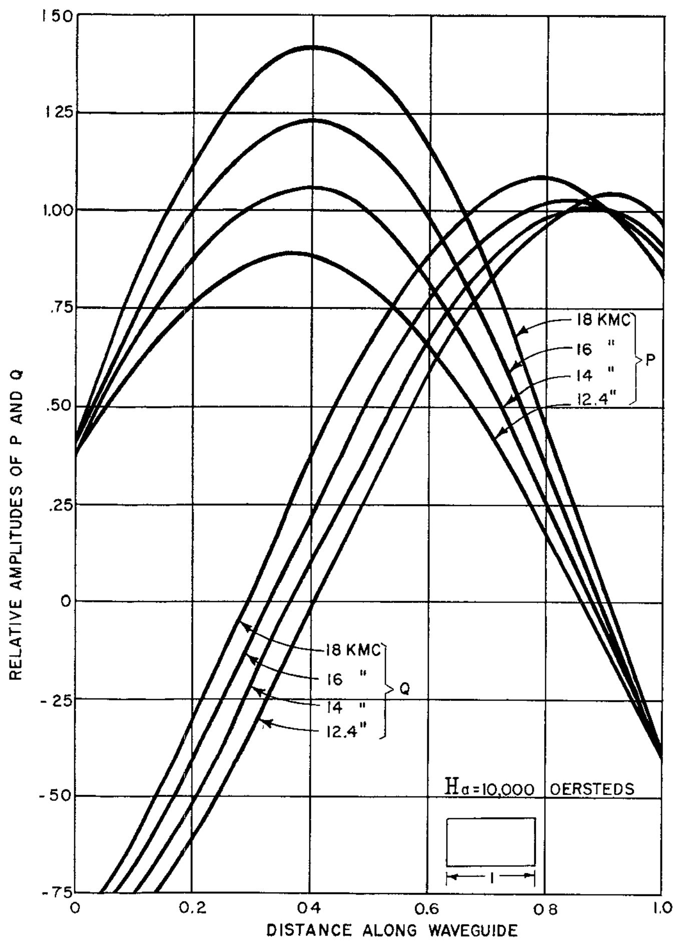


Fig. 5—Distribution of positive and negative elliptical polarization, in rectangular waveguide.

When the easy plane is oriented as in Fig. 4(c) (easy plane parallel to waveguide axis), then the location of the point of minimum absorption is given by the intersection of the Q curves with the horizontal zero axis. When the easy plane is oriented as in Fig. 4(d) (easy plane perpendicular to waveguide axis), the location of the point of minimum absorption is given by the intersection of the P curves with horizontal zero axis. It is apparent that the spread in the P curve intersections is far less than that of the Q curves, and also less than the corresponding spread for isotropic ferrites. This means that an isolator using planar ferrites with the easy plane perpendicular to the waveguide axis will be more broad-band than an isolator using isotropic ferrites, with respect to the shift in the location of the absorption minimum. Schlömann arrived at the same conclusion by a somewhat different approach.[11]

B. Phase Shifter

We will consider only the case of a thin slab in the geometry shown in Fig. 4(a), where the easy plane of magnetization is in the plane of the slab. We will also assume that the slab is thin enough so that perturbation theory can be applied. In this case, the phase shift will be proportional to the change in susceptibility, and we need therefore consider only the change in susceptibility as a function of applied field.

Using (11) we can write,

$$\chi_{xx} = 4\pi M_s \frac{H_0 + H_a + 4\pi M_s}{H_0(H_0 + H_a + 4\pi M_s) - H_i^2}.$$

Consider first that we are operating well below resonance. Then

$$\frac{\Delta\chi_{xx}}{\Delta H_0} = -\frac{4\pi M_s}{H_i^2}. \tag{29}$$

This is the same as for an isotropic ferrite with the same magnetization. However, H_0 will be limited to smaller values in the case of planar ferrites, and so the total possible phase shift is smaller. Thus, planar ferrites are not as desirable as isotropic ferrites in this manner of operation.

Eq. (29) assumes the material is saturated. If H_0 is zero, the material will be unsaturated, and the susceptibility will be zero, or close to zero. The application of a small field that is just enough to saturate the material will change the susceptibility to

$$\frac{4\pi M_s(H_a + 4\pi M_s)}{H_i^2}.$$

The ratio of this change of susceptibility to applied field will be much greater than that indicated in (29). This mode of operation may lead to more efficient phase shifters. The slab should be up against the narrow wall for this mode of operation.

Suppose now that we are operating well above resonance. Then

$$\chi_{xx} = \frac{4\pi M_s}{H_0}. \tag{30}$$

$\Delta\chi_{xx}/\Delta H_0$ will be much larger for planar ferrites than for isotropic ferrites, since H_0 is smaller in the case of planar ferrites. However, the more interesting comparison is between planar ferrites above resonance and isotropic ferrites below resonance. Let us call the ratio of change of susceptibility to change of applied field, the sensitivity of the phase shifter. The ratio of the sensitivity of the planar ferrite phase shifter to that of an isotropic ferrite phase shifter is equal to $(H_i/H_0)^2$, where H_i is equal to $\omega/|\gamma|$ and H_0 is the applied field for planar ferrites. Since H_i will be generally greater than H_0, the planar ferrite phase shifter will be more sensitive.

C. Switches

Planar ferrites may be advantageous for use in switches that operate on the principle of biasing the ferrite to resonance for the OFF position and removing the bias for the ON position, since the switching field is considerably smaller than for isotropic ferrites.

D. Harmonic Generation

Jepsen[12] shows that the efficiency of a ferrite in generating harmonics can be greatly enchanced if the ferrite is asymmetrical in the plane perpendicular to the direction of magnetization. He gives a formula quantitatively expressing this enhancement[13] and shows that it is proportional to the saturation magnetization multiplied by the difference in the demagnetizing factors in the plane perpendicular to the direction of magnetization. It has been noted above (Sec. II-B) that the anisotropy field behaves in the same way as does the inequality of demagnetizing factors. However, since the anisotropy field can greatly exceed the maximum field due to the inequality of demagnetizing factors, it is apparent that planar ferrites with high anisotropies are much more efficient harmonic generators than are isotropic ferrites.

Jepsen's equation[13] can be rewritten to take anisotropy into account. The result is

$$F = \frac{2}{\alpha}\left[H_a + M_s(N_z - N_x)\right] \cdot \frac{H_0 + H_a + M_x(N_z - N_y)}{(2H_0 + H_z + M_s(N_x + N_z - 2N_y))^2}. \tag{31}$$

F is the ratio of second-harmonic amplitude when there is asymmetry in the plane perpendicular to the direction of magnetization, to the amplitude when there is symmetry. The demagnetizing factors can be used to augment the anisotropy field to obtain efficient harmonic generation. Best results are obtained when $N_z = 4\pi$, $N_x = N_y = 0$. For this case,

$$F = \frac{2}{\alpha} \times \frac{G(H_0 + G)}{(2H_0 + G)^2}, \tag{32}$$

[11] E. Schlömann, "On the theory of the ferrite resonance isolator," IRE Trans. on Microwave Theory and Techniques, vol. 8, p. 199; March, 1960.

[12] R. L. Jepsen, "Harmonic Generation and Frequency Mixing in Ferromagnetic Insulators," Gordon McKay Lab., Harvard University, Cambridge, Mass., Scient. Rept. No. 15; 1958.

[13] *Ibid.*, eq. 5-31, p. 54.

where $G=(H_a+4\pi M_s)$. At a frequency of 70 kMc, and taking $G=30,000$ gauss (approximate values for Co_2Y), $\alpha=0.02$, F comes out to be 39. If G were 15,000 gauss, F would be approximately one half the above value.

VI. Experimental Data

The experimental data given in this section are intended primarily to illustrate the equations derived above. It is not intended to give comprehensive data on the properties of available planar ferrites or to detail design data on devices. As noted previously, all polycrystalline samples were oriented.

A. Measurement of Material Properties

1) *Preparation and Mounting of Samples:* Measurements were made using small spheres and wafers. The spheres were ground from rough-cut cubical samples by placing them in a glass tube perpendicular to a rapidly rotating abrasive wheel, as described by Carter, *et al.*[14] The diameter was approximately 40 mils. As might be expected from the magnetic anisotropy, there was also an anisotropy in the hardness, and the samples had ellipticities of approximately 10 per cent. This degree of ellipticity has only a very small effect and so it was disregarded.

The spheres were mounted on small polystyrene rods with the easy plane parallel to the axis of the rod. This was accomplished as follows: The sphere was placed on a glass platform and a permanent magnet placed underneath; the sphere thus automatically oriented itself so that the easy plane was perpendicular to the platform. A very small amount of vinyl adhesive was placed on the tip of the rod and the rod placed on top of the sphere. The magnet was removed and the rod could now pick up the sphere. The magnet was then placed on top of the platform and the rod, with the sphere on it, was placed underneath and remained there until the adhesive hardened.

The dimensions of the wafers were approximately $50\times50\times10$ mils. They were mounted on rods in the same way as were the spheres.

2) *Determination of Ferromagnetic Resonance Curves:* Ferromagnetic resonance curves were obtained for both the spheres and wafers. Measurements were generally made in a rectangular cavity, using the setup shown in Fig 6. The signal generators used were the Hewlett-Packard Models 626A and 628A, which contained accurate variable attenuators.

In the case of the sphere, the rod containing the ferrite was inserted in the opening on the broad face of the cavity. Since the sphere was oriented on the rod so that the easy plane was parallel to the rod, rotating the rod could bring the easy plane parallel or perpendicular to the RF magnetic field, as desired. The proper rotation of the rod for these conditions was determined by setting the biasing field to the value required for resonance and rotating the rod so as to bring the transmitted power to a minimum (easy plane parallel to RF field) or to a maximum (easy plane perpendicular to RF field).

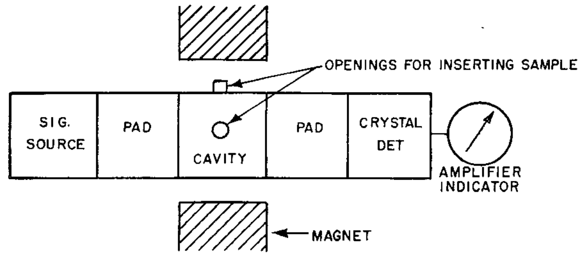


Fig. 6—Test setup for ferromagnetic resonance.

Two wafers were tested. One had the easy plane in the plane of the wafer. This was mounted on a rod with the plane of the wafer parallel to the rod. The rod was inserted through the opening on the broad face of the cavity and rotated so that the easy plane was parallel to the RF magnetic field. This was designated Case 1. The second wafer had the easy plane perpendicular to the plane of the wafer. This wafer was mounted on a rod with the plane of the wafer and the easy plane both parallel to the rod. This wafer was inserted in the opening on the broad face of the cavity and rotated so that the easy plane was parallel to the RF magnetic field. (See discussion above.) This was designated Case 2. The same wafer was then inserted in the opening on the narrow face of the cavity and rotated so that the easy plane was parallel to the biasing field. This was designated Case 3.

In some cases, where the losses at resonance were too high, the sample was measured by placing it at a position of maximum h field in a shorted transmission line, and by measuring the VSWR as a function of applied field. The shorted transmission line had openings for inserting the ferrite on the broad and narrow walls, as did the cavity.

Some of the curves obtained are shown in Figs. 7–9. The other curves are similar.

3) *Anisotropy Field:* The anisotropy was determined from the resonance curves of spheres or wafers. For spheres, the resonance condition given in (7) is used; *i.e.*, $H_0(H_0+H_a)=(\omega/\gamma)^2$. The wafer tested had the easy plane in the plane of the wafer and was oriented so that the RF field was parallel to the easy plane. Taking into account the geometry of the wafer, the resonance condition is given by $H_0[H_0+H_a+0.62(4\pi M_s)]=(\omega/\gamma)^2$. The value of γ was taken as 2.8 Mc/oersted.

[14] J. L. Carter, E. V. Edwards and I. Reingold, "Ferrite sphere grinding technique," *Rev. Sci. Instr.*, vol. 30, p. 946; October, 1959.

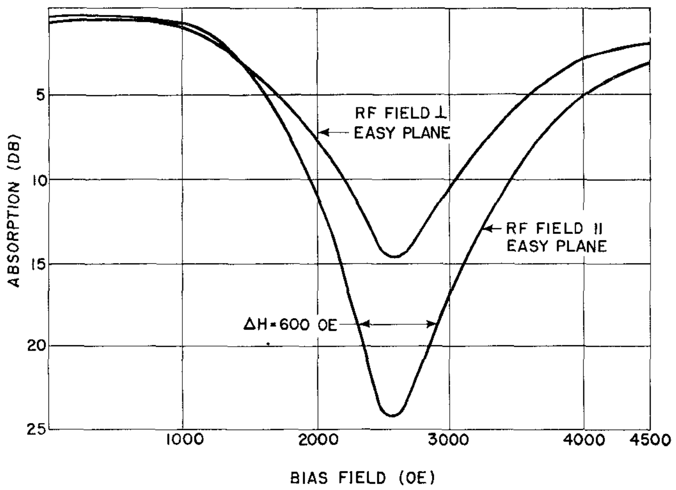


Fig. 7—Ferromagnetic resonance curves of Zn_2Y sphere at 14.9 kMc.

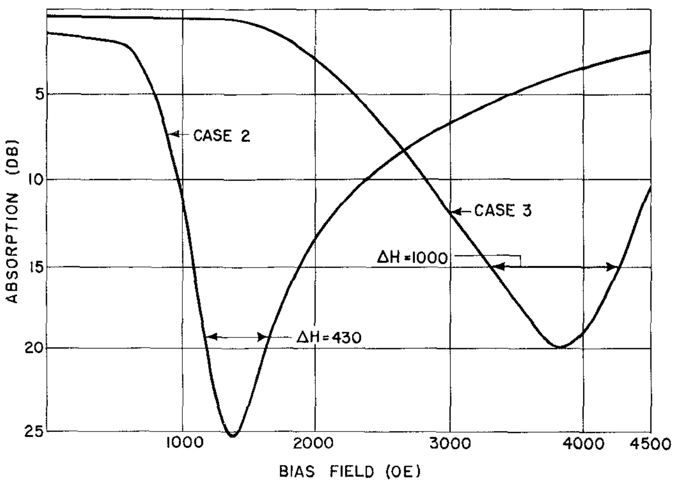


Fig. 8—Ferromagnetic resonance curves of wafers of Zn_2Y at 14.9 kMc.

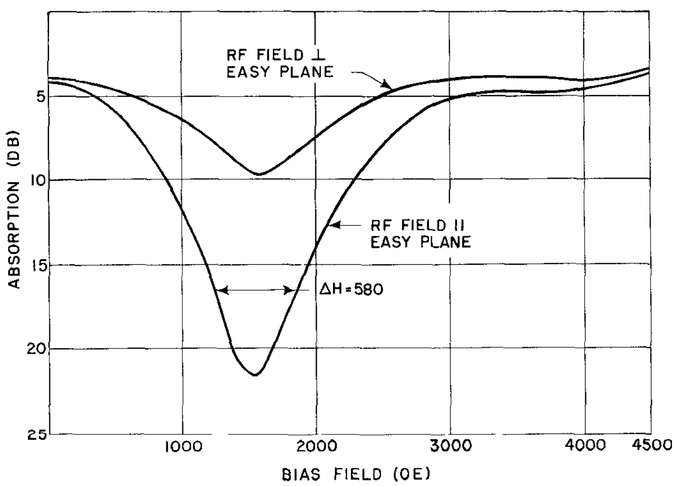


Fig. 9—Ferromagnetic resonance curves of Ni_2Y sphere at 14.9 kMc.

In Table II is a tabulation of the results obtained on polycrystalline samples. The Co_2Y was in the shape of a wafer; the others were spheres.

Single crystals of Zn_2Y were also measured.[15] Apparently the crystals were not perfect, since the measured values ranged from 8900 to 10,000 oersteds. Values of anisotropy field of Zn_2Y, Ni_2Y, and Co_2Y given by Smit and Wijn[1] are 9000, 14,000, and 28,000 oersteds, respectively. A torsion pendulum was used to obtain these values. Chemical compositions of the above materials are also given by Smit and Wijn.[1]

4) *Susceptibility Ratio:* Eq. 15 gives a theoretical formula for the ratio χ''_{xx}/χ''_{zz} at resonance. This formula was checked experimentally. The formula used to calculate the experimental ratio is given below:

$$\left(\frac{\chi''_{xx}}{\chi''_{zz}}\right)_{\text{exper}} = \frac{\left(\frac{V_0}{V_r}\right)_{xx} - 1}{\left(\frac{V_0}{V_r}\right)_{zz} - 1}, \tag{33}$$

where $20 \log_{10} (V_0/V_r)$ is equal to the decrease in transmitted output at resonance, expressed in decibels. The results are in Table III.

TABLE II

Material	Frequency (kMc)	H_a (oe)
Zn_2Y	10.8	9200
Zn_2Y	13.0	8400
Zn_2Y	14.9	8500
Zn_2Y	17.0	8400
Ni_2Y	13.0	17,100
Ni_2Y	14.9	17,200
Ni_2Y	17.0	16,700
Co_2Y	20.0	40,000

TABLE III

Material	Frequency (kMc)	Theoretical Ratio	Experimental Ratio	Theoretical/ Experimental
Zn_2Y (polyxtal)	10.8	7.6	4.3	1.76
Zn_2Y (polyxtal)	13.0	5.2	4.2	1.24
Zn_2Y (polyxtal)	14.9	4.3	3.5	1.23
Zn_2Y (polyxtal)	17.0	3.7	3.0	1.23
Ni_2Y (polyxtal)	14.9	12.3	6.9	1.78
Ni_2Y (polyxtal)	17.0	9.5	5.4	1.75
Zn_2Y (singlextal)	14.9	4.95	4.95	1.00
Zn_2Y (singlextal)	14.9	5.35	4.95	1.08

It is noted that the theoretical ratio is always equal to or larger than the experimental ratio. This to be expected, since any misalignment will tend to decrease χ''_{xx} and increase χ''_{zz}. The single crystals show much better agreement between theoretical and experimental ratios since they are much better aligned.

[15] The single crystals were grown at USADRDL, and are the results of the first attempt at preparing them. The work is continuing.

In the case of Ni_2Y there was a considerable amount of absorption not associated with the main resonance. This spurious absorption was excluded in calculating the experimental ratio (Fig. 9).

5) *Linewidth:* Linewidth was measured on the spheres and wafers. The results for the polycrystalline samples are tabulated in Table IV. In all cases the samples were aligned so that the easy plane was parallel to the RF magnetic field. The sphere and wafers of the Zn_2Y came from the same parent body.

It may be noted that the linewidth changes appreciably with frequency, particularly in the case of Ni_2Y. Also, there is a more than 2:1 change in the linewidth of a given wafer when measured per Case 2 or Case 3.

The linewidth of spheres of single crystals of Zn_2Y varied from 210 to over 300. The linewidth of a wafer of a single crystal Zn_2Y was 170 and 330 oersteds when measured per Case 2 and Case 3, respectively.

B. Measurement on Devices

1) *Phase Shifter:* A phase shifter was made using Zn_2Y. The sample was 1.5 inches long, 0.290 inch high, and 0.20 inch wide. The easy plane was parallel to the length of the sample. The sample was placed against the narrow wall of the waveguide. The test frequency was 15 kMc. The results are shown in Table V. Phase shift is with respect to phase at zero magnetic field.

2) *Isolator:* An isolator was made using Zn_2Y. The sample was 5/8 inch long and 0.05×0.05 inch in cross section. The easy plane was perpendicular to the waveguide axis. The optimum location of the sample for best front-to-back ratio over the full waveguide band was found to be such that the center of the cross section was 0.078 inch from the waveguide wall. The test results are shown in Table VI. The field was tuned for resonance in the reverse direction in each case.

A similar sample of Zn_2Y, but with the easy plane parallel to the waveguide axis, was also tested. It was not possible to find any location for the sample where the forward loss was low for the entire band. If the center of the sample was 0.177 inch from the narrow waveguide wall, the forward losses were low from 15 to 18 kMc, but high at the low frequencies. If the center of the sample was 0.204 inch from the narrow waveguide wall, the forward losses were low from 12.4 to 15 kMc, but high at the higher frequencies.

The above test data corroborate the discussion in Sec. V of this paper concerning the effect of the orientation of the easy plane on the broad-banding of the isolator.

TABLE IV

Material	Frequency (kMc)	Shape	Case	Line Width (oe)
Zn_2Y	10.8	Sphere	—	530
Zn_2Y	13.0	Sphere	—	570
Zn_2Y	14.9	Sphere	—	600
Zn_2Y	17.0	Sphere	—	600
Zn_2Y	14.9	Wafer	1	520
Zn_2Y	14.9	Wafer	2	430
Zn_2Y	14.9	Wafer	3	1000
Ni_2Y	13.0	Sphere	—	500
Ni_2Y	14.9	Sphere	—	580
Ni_2Y	17.0	Sphere	—	640
$Ni_{.2}Cu_{.2}Zn_{1.6}Y$	13.7	Wafer	1	325

TABLE V

Field (oe)	Phase Shift (degrees)
225	6
340	8
430	10
530	12

TABLE VI

Frequency (kMc)	Forward Loss (db)	Reverse Loss (db)	Field (oe)
12.4	0.10	1.55	1850
13	0.09	1.57	2050
14	0.08	1.60	2350
15	0.08	1.70	2650
16	0.10	1.82	3000
17	0.10	1.82	3350
18	0.10	1.95	3750

Acknowledgment

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